C-Curves

An alternative to the use of hyperbolic decline curves

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Introduction

In reservoir engineering, it is often useful to manipulate or create approximate production profiles. To be realistic, these profiles should honour the total amount of recoverable reserves and their shapes should match approximately whatever is typical for the type of reservoir considered. For fields showing exponential decline, it is straight-forward to construct a reasonable decline curve, using the relationship

Expected decline rate = (Production rate) / (Remaining technically recoverable reserves)

For fields showing hyperbolic decline, it is much more difficult to construct a reasonable relationship. The reason for this is that, in going from exponential to hyperbolic decline, the implicit strong relationship between profiles and recoverable reserves has been lost.

An alternative extension of exponential decline is given below, called the "C-curve." Derived during work on the Alba field (heavy oil, with underlying water, in a high permeability turbidite sand), this is based on extending the cumulative production equation, rather than the production rates equation, and yields the following relationship

$$Q_{oil} = R.\left[1 - \sqrt[b]{\frac{a}{a - 1 + e^{\left(a.b.Q_{liquid}/R\right)}}}\right]$$

where

Q_{oil} = cummulative oil production

R = technically recoverable reserves

Q_{liquid} = cummulative liquid production

a = constant (0.213 for Alba Extreme South)

b = constant (5.62 for Alba Extreme South)

Initial assumptions

The justification for exponential decline is usually quoted as the empirical observation that

$$\frac{dq_{oil}}{dt}\Big/ q_{oil} = -a$$

where

q_{oil} = oil production rate

t = time

a = constant.

This is then extended to hyperbolic decline in the form

$$\frac{dq_{oil}}{dt} \Big/ q_{oil} = -a.q_{oil}^{b}$$

However, examination of the physics of simple systems that exhibit exponential decline (such as the decay of radioactive particles, or production of a gas field under pure depletion) suggests that the fundamental driver is that the decay/production rate is proportional to the remaining population/reserves. In our case, this would be

$$\frac{dQ_{oil}}{dt} \Big/ (R - Q_{oil}) = -a$$

where

Q_{oil} = cummulative oil production

R = ultimate recovery

The "C-curve" method is to extend this relationship to the more general form

$$\frac{dQ_{oil}}{dt} \Big/ (R - Q_{oil}) = -(a + \beta(R - Q_{oil})^{b})$$

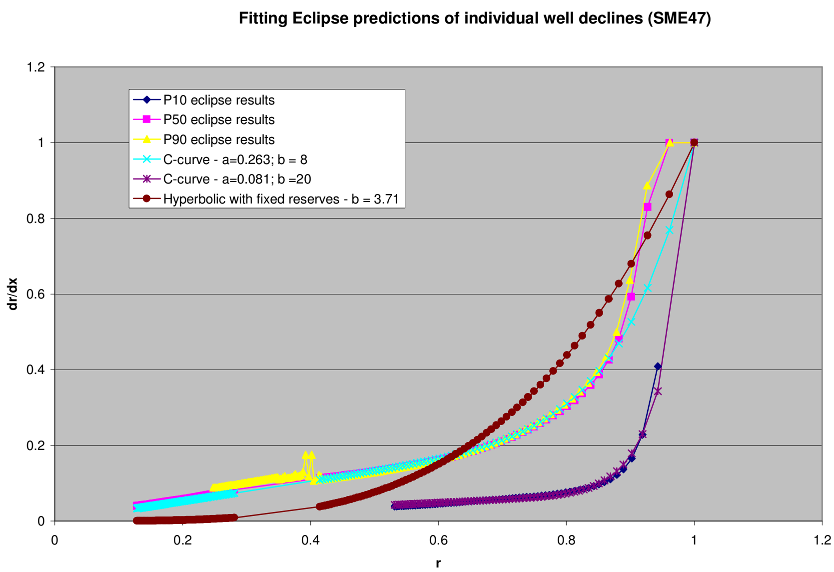
or, in a dimensionaless form

$$\frac{dr}{dx} \Big/ r = -(a + \beta r^{b})$$

where r = 1 – Q_{oil} / R

and x is any measure of field aging – very Q_{liquid} / R

The plot below illustrates how this relationship is sufficiently general to give a good match to typical individual well declines in Alba, as predicted in reservoir simulation.



The equivalent relationship in hyperbolic decline is

$$\frac{dr}{dx}\Big/ r = -\beta r^{b}$$

It can be seen from the plot that hyperbolic decline does not give a good fit, at least not with the same figure for ultimate reserves. On Alba, it was found that hyperbolic decline gave reasonable fits only with high exponents (i.e. close to harmonic decline) and enormously high reserves figures (e.g. 1100 million bbls for Alba Extreme South, compared to a STOIIP of 320 million bbls).

The key idea behind this approach is that it is the dr/dx – r relationship that matters for creating life-of-well or life-of-field production profiles. The exact form of the relationship chosen does not matter much, providing it is sufficiently general to fit the shape of decline as observed in reality or as predicted in Eclipse. The C-curve relationship was chosen so as to be easily solvable to yield formulae that can be easily used and manipulated.

Solving for cumulative oil

Starting from the initial equation

$$\frac{dr}{dx}\Big/ r = -(a + \beta r^{b}) \qquad \text{(A)}$$

the variables can be split as follows

$$\frac{1}{r\left(a+\beta r^{b}\right)}dr=-dx \qquad \text{(B)}$$

Integrating both sides gives

$$\frac{1}{\mathrm{ab}}\ln\left(\frac{r^{b}}{a+\beta r^{b}}\right)=-(x+\alpha) \qquad \text{(C)}$$

where α is a constant.

[Proof of integration of left hand side –

$$\frac{d}{dr}\left(\frac{1}{\mathrm{ab}}\ln\left(\frac{r^{b}}{a+\beta r^{b}}\right)\right)=\frac{d}{dr}\left(\frac{1}{\mathrm{ab}}\ln\left(\frac{1}{ar^{-b}+\beta}\right)\right)$$

$$=\frac{1}{ab}\left(ar^{-b}+\beta\right).\frac{-1}{\left(ar^{-b}+\beta\right)^{2}}.\left(-abr^{-n-1}\right)=\frac{1}{r\left(a+\beta r^{n}\right)}$$

.

]

Solving equation (C) for r gives

$$r=\sqrt[b]{\frac{a}{e^{ab(x+\alpha)}-\beta}}$$

The usual boundary conditions include

a) production starts with dry oil i.e. dr/dx = 1 when r =1
b) At the start of production (i.e. when r =1) x = 0

These boundary conditions allow us to express the α and β in terms of other variables, as follows

Condition (a) implies (from equation (A))

-1 = -(a + β)

i.e.

β = 1 – a

Applying condition (b) to equation (B) gives

$$\frac{1}{ab}\ln\left(\frac{1}{a+(1-a)1^{b}}\right)=-(0+\alpha)$$

i.e.

α = 0

Applying these values of α and β to equation (C) gives

$$r = \sqrt[b]{\frac{a}{e^{abx} + a - 1}}$$

Changing from r and x to R, Q_{liquid} and Q_{oil} gives

$$Q_{oil} = R.\left[1 - \sqrt[b]{\frac{a}{a - 1 + e^{\left(a.b.Q_{liquid}/R\right)}}}\right]$$

Interestingly, it was found that the r, x formulation could also be used in rather different way. If x was taken to be the number of wells drilled on the field, and r to be the fraction of movable oil remaining at the end of field life after the x wells had been produced to very high water-cuts, then

$$\frac{dr}{dx} = -\beta r^2$$

was found to give a very good match to a set of simulator runs with varying numbers of wells, as is illustrated in the two plots below.